MTH 234 - Quiz 5

Due 29 June at the beginning of class

You may work together on solving these problems, but what you turn in must be your own work written in your own words; copying another person's work is not allowed. Please present your work in a clear and organized fashion, and staple your pages (if needed). Late quizzes will receive at most 75% credit if they are turned in by the middle of class; after that, no more quizzes will be accepted.

1. (10 points) A torus with small radius r and large radius R ($r < R$) is described by the equation

$$r^2 = z^2 + \left(R - \sqrt{x^2 + y^2}\right)^2$$

   Find a parameterization of this object, and use it to show that the surface area is exactly $4\pi^2 Rr$.

   Hint: Draw the picture in the xz-plane; a natural choice of parameters is to take the angles inside the large and small circles. Look at a circle centered at the point $(R, 0)$ with radius r; use polar coordinates for inspiration. Compute the x coordinate of a point based on the angle in the small circle.

2. (10 points) Verify that Stokes' Theorem holds for the vector field $\vec{F}(x, y, z) = x^2\vec{i} + y^2\vec{j} + z^2\vec{k}$, where S is the part of the paraboloid $z = 1 - x^2 - y^2$ that lies above the xy-plane and S has upward orientation. That is, compute the integrals

$$\int_C \vec{F} \cdot d\vec{R} \quad \text{and} \quad \iint_S \operatorname{curl} \vec{F} \cdot d\vec{S}$$

   (where C is chosen appropriately) separately and check they are the same. Include a sketch of the the surface S and the curve C.